A Generalized Attention Deficit Disorder Chaotic Model With Soboleva Hyperbolic Tangent Functions

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***Abstract*—This work studies the problem of modeling Attention Deficit Disorder (ADD) using a previously reported one dimensional map, but by replacing its two hyperbolic tangent activation functions, with Soboleva hyperbolic tangent ones. This adds four new control parameters to the system, which significantly enhances its modeling freedom. Afterwards, the effect of these parameters on the model is studied, using tools of nonlinear analysis, like bifurcation, Lyapunov exponent, and phase diagrams. The emergence of chaos is prevalent, indicating the sensitivity of the model to external excitations. Numerous phenomena are also observed, like crisis, antimonotonicity, and shrimps. This new model can help delve deeper into the emergence of chaos in behavioral disorders.**

***Index Terms*—Attention deficit disorder, chaos, Soboleva activation function, neural network**

Chaos theory is one of the great scientific discoveries of the 20th century. Chaos as a phenomenon appears in many different areas of science [1]. One area is biology, where various biological processes are modeled using chaotic systems [2], [3]. In this area, chaos appears in the models of various disorders, such as neurodegenerative diseases [4], attention deficit disorder (ADD) [5]–[9], migraine [10], seizure [11], epilepsy [12], bipolar disorder [13], coma [14], and more [15].

The rise of chaotic models in these areas results from chaotic systems being characterized by very complex behavior, ranging from stable solutions (fixed points, cycles) to unstable (chaotic) solutions. In addition, an important phenomenon that occurs in chaotic systems and serves to justify these models is intermittency [5]. Intermittency is characterized by the switching of the system between stable and chaotic behavior. Such behavior can model the state of a person with behavioral disorders, the focus of this work.

Chaotic models that describe the issues mentioned, often reflect a simple neural network with two neurons. These neurons are activated using a so-called activation function, with the $\tanh$ being the common choice. Such a defined neural network model is characterized by chaotic phenomena, including intermittency [5], among others.

From the above short introduction to the topic of chaos applications in modeling behavioral conditions, it is clear that this is a current and significant topic. In light of the work [5] and the chaotic model presented therein for attention deficit disorder (ADD), this work proposes a significant modification. This modification is the replacement of the activation function $\tanh$ in the model [5] with its more flexible and controllable counterpart, the Soboleva hyperbolic tangent function (smht) [16], [17]. The smht function, with its four control parameters, offers a level of control over the model's behavior that was previously unattainable. This increased fitness control promises a more precise and accurate model, while also enabling more complex model dynamics to arise. An extensive analysis of the new model is performed with respect to the newly introduced parameters. The analysis reveals a plethora of chaotic phenomena appearing, indicating a higher complexity in the model. This can be a starting point for a more in depth analysis of the chaotic dynamics present in the ADD model, as well as other models of neurological disorders.

The rest of the work is structured as follows: Section I presents the activation function. Section II presents the modified dynamical model for ADD. In Section III, the dynamical

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analysis is performed. Section IV discusses the results. Section V concludes with a discussion on future studies.

I. THE SOBOLEVA HYPERBOLIC TANGENT

The Soboleva modified hyperbolic tangent function is described as follows [16], [17]:

$$\mathrm{smht}(x) = \frac{e^{ax} - e^{-bx}}{e^{cx} + e^{-dx}}, \tag{1}$$

where $x \in \mathbb{R}$, and $a, b, c, d \in \mathbb{R}$ are control parameters, with $a \leq c$, $b \leq d$. The control parameters can be appropriately tuned to control the shape of the function. Clearly, (1) is equal to the classic $\tanh$ function for $a = b = c = d = 1$. Thus, (1) is a generalization of the classic hyperbolic tangent. An example for different parameter values is shown in Fig. 1. When the control parameters are unequal, the function's symmetry around the origin is broken.

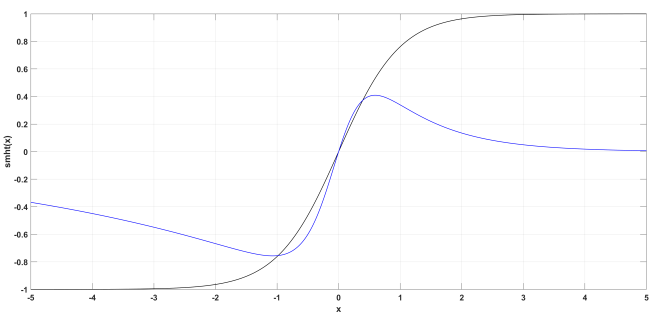

Fig. 1. The Soboleva activation function (1) for $a = b = c = d = 1$ (black, the classic $\tanh$) and $a = 2$, $b = 0.8$, $c = 3$, $d = 1$ (blue).

II. THE PROPOSED MODEL FOR ATTENTION DEFICIT DISORDER (ADD)

The ADD model considered in this work is the following, introduced in [5]. The model consists of two neurons that process the input information. It is described as follows:

$$x_i = B\tanh(w_1 x_{i-1}) - A\tanh(w_2 x_{i-1}), \tag{2}$$

where x_i is the information signal, and the $\tanh$ functions are the activation functions of two neurons, responsible to transform the information signal propagating across the brain. The positive term represents the excitatory brain action and the negative term the inhibitory action to the information signal. The parameters A, B, control the magnitude of each activation function, while w_1, w_2 control the amplification of the signal into each activation function. Chaotic behavior was interpreted here as switches in the attention level over small time intervals.

In the modification proposed here, the activation functions are replaced by Soboleva modified activation functions:

$$x_i = B\mathrm{smht}(w_1 x_{i-1}) - A\mathrm{smht}(w_2 x_{i-1}), \tag{3}$$

where each activation function is of the form (1), with control parameters a, b, c, d. Thus, there are four new control parameters introduced, which bring a significant improvement to the degrees of freedom of the model.

III. DYNAMICAL ANALYSIS

In this section, the dynamical behavior of the proposed model is studied. As in [5], the parameter A is of interest, along with the variations on a, b, c, d. The rest of the parameters are set to $B = 5.821$, $w_1 = 1.487$, $w_2 = 0.2223$, which were chosen in [5]. Also, the initial condition is $x_0 = 0.1$.

A. Original Model

The model (2) exhibited periodic and chaotic behavior, with rich transitions in between these two states. A bifurcation diagram for (2) is shown in Fig. 2 (black). Several phenomena can be identified. First, a period-doubling route to chaos is observed, as A increases from 5 to around 8.5. This is followed by a crisis phenomenon, where the attractor abruptly expands for A around 9.5. Afterwards, the model exhibits more crisis phenomena, where it abruptly exits chaotic behavior for A around 12.5, followed by period doubling route to chaos. The reverse phenomenon appears near $A = 20$, where the attractor's size reduces, and a transition out of chaos through period halving route appears, which is followed by another crisis entering chaos for A around 21.2. Afterwards, another period halving transition out of chaos is observed.

The transitions in and out of chaos are evident in Fig. 3, that depicts the values of the Lyapunov exponent (LE) of (2) for different parameter pairs (A, B). A positive LE indicates chaotic behavior, while a negative indicates a non-chaotic one. Here, dense regions of transitioning in and out of chaos appear (at the boundaries between the black color (periodicity), and other colors (chaos)), indicating the system's sensitivity to parameter changes. Next, it will be seen that the modified system exhibits similar, but more complex phenomena.

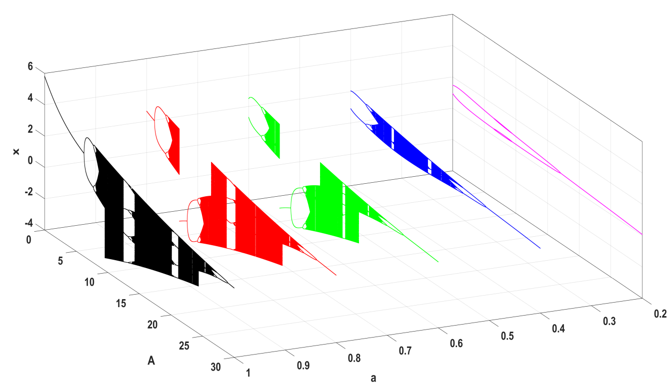

Fig. 2. Bifurcation diagrams of the system (3) with respect to parameter A, for different values of a, for $b = c = d = 1$, depicted in the 3d space.

B. Proposed Model

Due to the limited space, focus will be given on parameters a, b for the proposed model (3).

1) Changes to parameter a: To study the behavior with respect to changing a, several bifurcation diagrams with respect to parameter A are shown in Fig. 2, for different values of

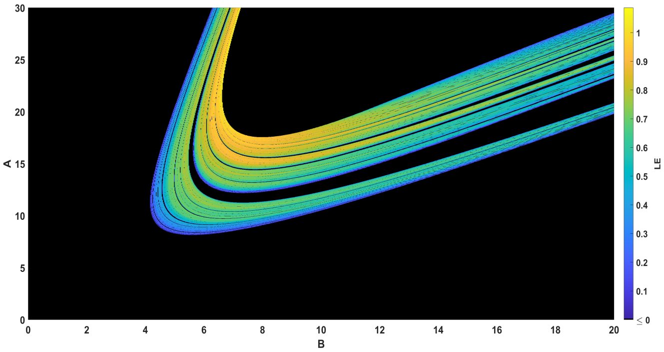

Fig. 3. Lyapunov exponent values for parameter pairs (A, B) for system (2).

a. Compared to the original model (black), it is evident that changing the parameter a changes the behavior significantly.

Although a general trend for starting from periodicity for low values of A and ending again in periodicity for higher values of A appears in all the graphs, the behavior in-between varies between graphs. The chaotic and non-chaotic regions appear in different parametric ranges of A. More crisis phenomena are observed, especially for $a = 0.8, 0.6$, where the attractor abruptly transitions to a different region in the state space. This phenomenon could be indicative of possible coexisting attractors present in the system. Coexisting attractors are also present in the original system, and this was noted in [18], but this jumping phenomenon is not observed in the original bifurcation diagram of Fig. 2 (black).

Moreover, as a decreases, the attractor's size decreases, and for $a = 0.2$, chaotic behavior is completely absent from the diagram. This is indicative that lower a values can have a suppressive effect on chaotic behavior. Another notable phenomenon is the emergence of antimonotonicity. This refers to the phenomenon where the system enters into chaotic behavior through a period-doubling route as the control parameter increases, and then exits it following a reverse period-halving route. This appears for $a = 0.8$, shown in Fig. 2 (red). Antimonotonicity also appears for $a = 0.2$ in Fig. 2 (magenta), for a small periodic transition.

2) Changes to parameter b: Fig. 4 depicts several bifurcation diagrams with respect to A, for different values of b. Similar phenomena to when changing a can be observed. There are crisis phenomena present, where the attractor jumps on a different position in the state space, for $b = 0.8, 0.6, 0.4$. Another fact is that the attractor shrinks in size as b decreases, although not as strongly as in the case of changing a.

3) Changes to both parameters a and b: In the above graphs, changes to only a single Soboleva control parameter was considered, to understand how each individual parameter can affect the system's behavior. Naturally, changing multiple parameters gives rise to more complex dynamical behaviors. For example, in contrast to the default case shown in Fig. 3, Fig. 5 depicts the LE values for parameter pairs (A, B) for $a = 2$, $b = 0.8$, $c = 3$, $d = 1$. The graph is significantly different from the previous one. There are multiple wide

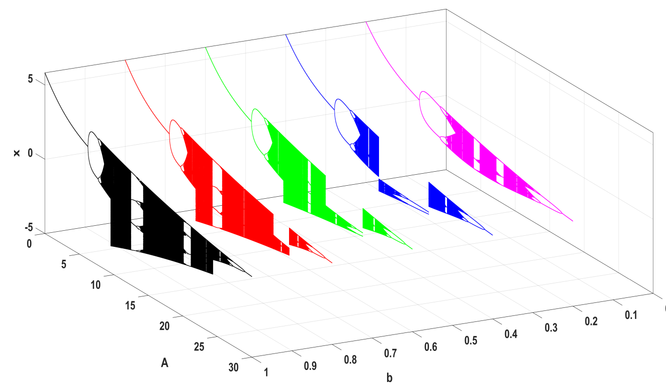

Fig. 4. Bifurcation diagrams of the system (3) with respect to parameter A, for different values of b, for $a = c = d = 1$, depicted in the 3d space.

regions of non-chaotic behavior present. But more importantly, the transitions between chaotic and non-chaotic regions are much more interplexed, indicating that for specific parametric ranges, the system exhibits a very high sensitivity to changes in the parameter pairs (A, B). These dense regions between chaos and order can be interpreted as an inability of the system to adapt to small changes in its parameters, which is indicative of poor responsive behavior.

To further explore the effects of changing multiple Soboleva parameters, Fig. 6 shows LE values for the Soboleva parameter pairs (a, b), under parameters $A = 13$, $B = 5.821$, $c = d = 1$. The graph reveals certain symmetries in the parameter pairs around the region $(a, b) \in (0.7, 1) \times (0.7, 1)$. The behavior is intricate, as a complex shape appears with multiple transitions in and out of chaos. Small shrimp regions also appear here. As each of the parameters a, b decrease, the LE reduces, until eventually the system enters into a non-chaotic regime, which is sustained for all parameter pairs.

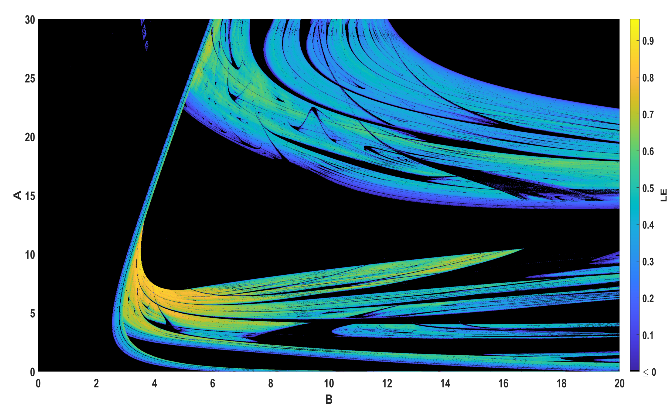

Fig. 5. Lyapunov exponent values for parameter pairs (A, B) for the system (3), for $a = 2$, $b = 0.8$, $c = 3$, $d = 1$.

IV. DISCUSSION

From the simulations performed, it is evident that the introduction of four additional parameters from the Soboleva

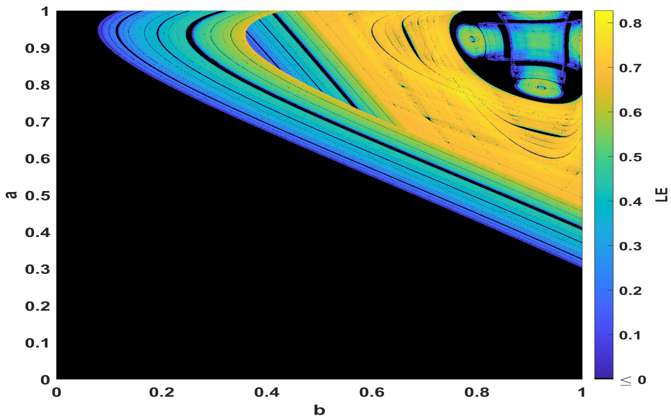

Fig. 6. Lyapunov exponent values for parameter pairs (a, b) for the system (3), for $A = 13$, $B = 5.821$, $c = d = 1$.

hyperbolic tangent generalizes the system, making it capable of exhibiting a richer dynamical behavior. What is notable, is that for many parametric ranges, the model exhibits dense transitions in and out of chaos. This was present in the original system as well, but is more emphasized in the modified model. This indicates an inability of the system to adapt to small parameter perturbations. The self similar shrimp shapes also appear. Crisis phenomena are observed, where the attractor either expands abruptly, or changes location. The introduction of four new parameters increases the modeling freedom of the system, which can expand its adaptability in modeling other neurological phenomena, like autistic behavior. Of course, an interesting question is how the changes in the shape of each activation function can be interpreted neurologically. If this can be identified, then future control techniques can direct their effect on changing each of these control parameters, to improve the model's behavior.

Although this is a simplified model of theoretical interest, it is interesting to see that despite its simplicity, its dynamics are rich. It is evident from the graphs that the model is very sensitive to parameter changes. This means that robust control techniques should be applied to control its behavior, especially considering that these parameters could be time varying.

V. CONCLUSIONS

In this work, a generalization of the attention deficit disorder model from [5] was proposed, by replacing the traditional $\tanh$ functions with Soboleva hyperbolic tangent functions. This introduced four new control parameters, which significantly increases the model's potential to exhibit more dynamical phenomena. An extensive analysis with respect to the Soboleva parameters indeed revealed that the system has rich dynamics.

There are several goals set for future studies. First, a full scale analysis with respect to all of the four control parameters must be performed. Secondly, the existence of coexisting attractors should be explored. Thirdly, the effect of an attenuator feedback, to model imperfect perception of input information must be considered. Fourth, although this is a theoretical model, it would be interesting to see if the introduction of the new control parameters could help in fitting the model to real measurements. Due to having eight parameters overall, machine learning techniques could be applied to perform the fitting to real data. Moreover, different variations of the model can be considered. Specifically, it is of interest to model the effect of memory, either through fractional modeling, or through the use of memristive elements. Overall, the present work can be the starting point for several new studies.

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